ON THE EVALUATION OF OPTIMAL PARAMETERS OF A DRY-FRICTION DAMPER

(K RASCHETU OPTIMAL'NYKH PARAMETROV DEMPFERA SUKHOGO TRENIIA)

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M.I. FEIGIN
(Gor'ki)

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The evaluation of the optimal parameters of a dry-friction damper for a model with half a degree of freedom under the assumption that the damping system undergoes a sinusoidal motion was given in [1-3].

In this paper the author gives an evaluation of the optimal parameters of a dry-friction damper and computes the energy dissipated by the damper during one period of oscillation (energy capacity of the damper) for a model which is a nonautonomous system with one-and-a-half degrees of freedom. A comparison of the energy capacity at resonance frequencies for various values of the frictional force shows that to an optimal adjustment of the damper there corresponds a minimum of the dissipated energy.

1. We shall make use of the results of the investigation of a dry-friction damper given in [4] by the method of point transformations. Figure 1 shows the mathematical model which was used in the indicated work. It consists of an elastically (with the elastic constant k) attached mass M, which is acted upon by an external force $F \sin \Omega t$. The interaction of the mass m of the damper with the mass M is due entirely to the force of dry [Coulomb] friction Φ. Let $x = 0$ correspond to the undeformed state of the spring. Excluding from our consideration the simultaneous motion of the masses without relative sliding, we have the following equation of motion for the model:

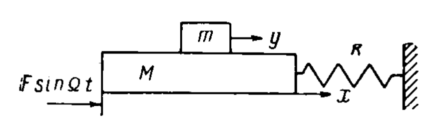

Fig. 1.

$$M\ddot{x} + kx = F \sin \Omega t \pm \Phi, \quad m\ddot{y} = \mp \Phi \tag{1}$$

Let us now introduce dimensionless variables in Equation (1)

$$\xi = xM\Omega^2 / F, \qquad \eta = yM\Omega^2 / F, \qquad \tau = \Omega t \tag{2}$$

Then we obtain the equations

$$\ddot{\xi} + \omega^2\xi = \sin\tau + \beta \operatorname{sgn}(\dot{\eta} - \dot{\xi}), \qquad \mu\ddot{\eta} = -\beta \operatorname{sgn}(\dot{\eta} - \dot{\xi}) \tag{3}$$

in which the dimensionless parameters are given by

$$\beta = \Phi / F, \qquad \omega^2 = k / M\Omega^2, \qquad \mu = m / M. \tag{4}$$

If the solution $\xi(\tau)$ of Equation (3) which corresponds to a motion of period 2π is expanded into a Fourier series, then one obtains an expression for the first and higher harmonics:

$$\Psi_1^2 = \frac{1}{(\omega^2 - 1)^2}\left\{1 + \frac{16\beta^2}{\pi^2}\left[1 + \frac{\pi(\omega^2 - 1)}{2\omega}\left(\frac{\pi\omega}{2\mu} + \tan\frac{\pi\omega}{2}\right)\right]\right\} \tag{5}$$

$$\Psi_n^2 = \frac{16\beta^2}{\pi^2 n^2 (\omega^2 - n^2)^2} \qquad (n = 3, 5, 7, \ldots) \tag{6}$$

Expressions (5) and (6) are valid in the region where the considered motion exists and is stable. This region includes the values of the parameters β, ω, and μ satisfying the inequality

$$\beta^2\left[\frac{\omega^2(1 + \mu)^2}{\mu^2} + \left(\frac{\pi\omega}{2\mu} + \tan\frac{\pi\omega}{2}\right)^2\right] < \frac{\omega^2}{(\omega^2 - 1)^2} \tag{7}$$

We shall determine the optimal parameters of the damper by investigating the amplitude of the first harmonic, and by estimating the measure of the nonsinusoidal nature of the function $\xi(\tau)$ on the basis of the distortion factor

$$\varkappa^2 = \frac{1}{\Psi_1^2}\sum_3^\infty \Psi_n^2 \quad .$$

If one fixes the value of the frictional force β and the mass μ of the damper, then the dependence of the amplitude of the damped oscillations on the frequency,. i.e. the function $\Psi_1(\omega)$, will be given by a curve with a maximum at some resonance frequency $\omega = \omega^*$. If one varies β, then ω^* changes and so does the amplitude $\Psi_1(\omega^*)$ for the resonance frequency. To the optimal frictional force β_0 there corresponds the minimum amplitude $\Psi_1(\omega^*)$ min. In the upper parts of Fig. 2 (μ = 0.1), and of Fig. 3 (μ = 0.5), there are presented two families of the dependences $\Psi_1(\omega)$ for various values of β. The figures show that as β increases, the maximal amplitude $\Psi_1(\omega^*)$ of the first harmonic first decreases and then increases. The value β_0 corresponds to the case when

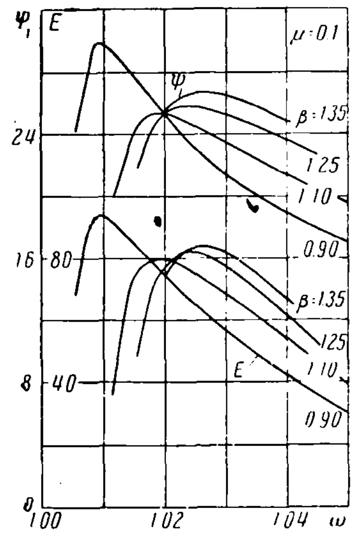

Fig. 2.

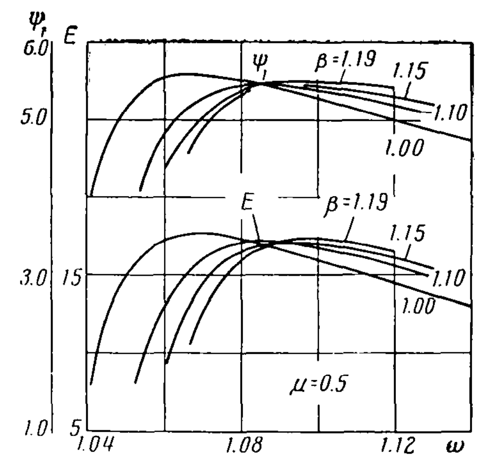

Fig. 3.

the maximum of the curve coincides with the common point of intersection of the curves of the family. The coordinate ω_0 of the mentioned common point is equal to the resonance frequency of the system with the optimally adjusted damper. From Expression (5), it follows directly that ω_0 is given by the equation

$$1 + \frac{\pi(\omega_0^2 - 1)}{2\omega_0}\left(\frac{\pi\omega_0}{2\mu} + \tan\frac{\pi\omega_0}{2}\right) = 0 \tag{8}$$

The optimal frictional force β_0 can be found from the condition that the curve $\Psi_1(\omega)$ has a maximum at the point ω_0. By differentiating $\Psi_1(\omega)$, and making a number of transformations with the aid of (8), we arrive at the following equation:

$$\beta_0 = \omega_0\left[(\omega_0^2 - 1)^2\left(\frac{1+\mu}{\mu} + \tan^2\frac{\pi\omega_0}{2}\right) - \frac{4}{\pi^2}(\omega_0^2 + 1)\right]^{-1/2} \tag{9}$$

Using (7) one can convince oneself that for practical, applicable values of μ the quantities ω_0 and β_0 will belong to the region of the periodic motion under consideration.

The optimal amplitude of the first harmonic at resonance frequency ω_0 obtained from Equation (5) is equal to

$$\Psi_1^\circ = \frac{1}{\omega_0^2 - 1} \tag{10}$$

An estimate of the nonsinusoidal nature of the function $\xi(\tau)$ at β_0 and ω_0 yields

$$\varkappa^2 = 2.83\ 10^{-5} \quad \text{for } \mu = 0.1, \qquad \varkappa^2 = 6.59\ 10^{-4} \quad \text{for } \mu = 0.5$$

The evaluation of the optimal parameters on the basis of the first harmonic has thus been shown to be justified.

In the case when $\mu \ll 1$, one can expand Expressions (8) to (10) in power series in terms of $\Delta\omega = \omega - 1$, and one may neglect terms containing $\Delta\omega$ to a degree higher than the first. One thus obtains

$$\omega_0 \approx 1 + \frac{2\mu}{\pi^2}, \qquad \beta_0 \approx \frac{\pi}{\sqrt{8}}, \qquad \Psi_1{}^\circ \approx \frac{\pi^2}{4\mu} \tag{11}$$

In accordance with the results of [4], the energy capacity of the damper in the case under consideration is equal to

$$E = \frac{4\beta}{\omega(\omega^2 - 1)}\left[\omega^2 + \beta^2(\omega^2 - 1)^2\left(\frac{\pi\omega}{2\mu} + \tan\frac{\pi\omega}{2}\right)^2\right]^{1/2} \tag{12}$$

In the lower part of Figs. 2 and 3 there are given the families of curves $E(\beta, \omega, \mu)$ computed on the basis of Formulas (12) for the values $\mu = 0.1$ and $\mu = 0.5$. Each of the curves attains a maximum value E^* at the same resonance frequency ω^* as does $\psi_1(\omega)$. As β changes, so do ω^* and E^*. A comparison of the energy capacity for various frequencies corresponding to different values of β shows that for the optimal adjustment of the damper β_0 corresponds to the minimum dissipated energy.

2. The last result has the following physical meaning. If the damped oscillations are nearly sinusoidal, then the work done by the disturbing force on the resonance frequency during one period of oscillation is approximately equal to $\pi F x_0$, where F is the amplitude of the disturbing force, while x_0 is the amplitude of the damped oscillations. During the stabilized forced oscillations, this work is equal to the energy capacity of the damper. This shows that for the minimum value of the resonance amplitude x_0, the energy dissipated by the damper will also be a minimum.

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